

**Reg Kulperger**  
**is**  
**Responsible for All of My Problems**

W. John Braun

University of British Columbia

Time Series, Spatial Process and Asymptotic Methods  
30 May 2019

## **Outline**

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### **The Facts About Reg Kulperger**

- **as I learned about them from Reg**
- **and as I remember them**

### **Consequences**

## **Early Life and Education**

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**Reg was born in the last century in the GBA.**

**He was educated in Brantford, Ontario.**

**He graduated from High School as an Ontario Scholar during the rise to fame of one of Canada's more charismatic politicians so that unfortunately,**

**Kulpergermania is not a thing.**

## **Early Life and Education**

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**He began studies at the University of Waterloo in the mathematics program.**

**Notable classmates: Nancy Reid, Francis Zwiars, and many others.**

**Reg enjoyed all aspects of mathematics: algebra, analysis, measure theory, topology, ... and probability and statistics.**

## **Early Life and Education**

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**His Masters supervisor was Mary Thompson, who fostered his deep appreciation for stochastic processes and statistical inference.**

**He left for Carleton University, to pursue a PhD with Don Dawson.**

# PhD Cover Page

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CANADIAN THESES ON MICROFICHE / THÈSES CANADIENNES SUR MICROFICHE

NAME OF AUTHOR/NOM DE L'AUTEUR REGINALD J. KULPERGER

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SIMPLE BRANCHING DIFFUSION  
PROCESS

UNIVERSITY/UNIVERSITÉ CARLETON UNIVERSITY

DEGREE FOR WHICH THESIS WAS PRESENTED/  
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YEAR THIS DEGREE CONFERRED/ANNÉE D'OBTENTION DE CE DEGRÉ 1978

NAME OF SUPERVISOR/NOM DU DIRECTEUR DE THÈSE DR. D. DAWSON

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DATE/DATE 14 June 1978 SIGNED/SIGNÉ Reginald J. Kulperger

PERMANENT ADDRESS/RÉSIDENCE FIXE 47 ADMIRAL ROAD, BRANTFORD, ONTARIO N3R 1L5

93 (11-78)

Question for Discussion:  
What is Don Dawson's  
first initial?

## Abstract

In a simple biological population, for example of plankton, or of mutant genes introduced into a population of "normals", individuals migrate, give birth and die in some domain  $D \subseteq \mathbb{R}^d$ . A branching diffusion process is a mathematical model which describes this stochastic evolutionary behaviour. A particular model known as a simple branching diffusion process is considered here. The model is a simple branching process, with Brownian movement or migration added, in which individuals either die or split into two individuals. In addition, immigration is assumed to take place. Let  $\theta$  be the parameter of the process, consisting of the birth and death rates, the diffusion rate, and the immigration rate.

The main purpose of this study is to consider the problem of estimating the parameter  $\theta$  of the above mentioned process.

Let  $\{X_t(x): x \in \mathbb{R}^d, t \geq 0\}$  be the stochastic evolutionary process. If  $0 \leq \tau_1 < \dots < \tau_L$  are taken to be fixed times, then  $\{X(x) = (X_{\tau_1}(x), \dots, X_{\tau_L}(x)): x \in \mathbb{R}^d\}$  is an L-dimensional point process on  $\mathbb{R}^d$ . Such an induced point process is called an L time slice version of the stochastic evolutionary process. For the simple branching diffusion immigration process, the parameter of an L time slice version is  $(\theta, \tau_1, \dots, \tau_L)$ . Estimating this parameter yields an estimate of  $\theta$ .

## Themes:

- Birth and Death Process (with Immigration)
- Diffusion
- Parameter Estimation
- Count Process

Now consider a particular  $L$  time slice version of the simple branching diffusion immigration process, with times  $0 < \tau_1 < \dots < \tau_L$ . Let  $N_\tau(A)$  be the number of individuals in  $A$  at time  $\tau$ . For the two sets  $A$  and  $B \subseteq \mathbb{R}^d$ , the random variables  $N_{\tau_i}(A)$  and  $N_{\tau_i}(B)$  are not independent. It is shown, however, that this process satisfies some Brillinger-type mixing conditions. Various central limit theorems (CLT's) are given for Brillinger mixing point processes. As a corollary, a CLT is immediately obtained for  $(N_{\tau_1}(A), \dots, N_{\tau_L}(A))$  as  $A \uparrow \mathbb{R}^d$ .

Using CLT's for Fourier transforms of Brillinger mixing point processes, an asymptotic likelihood function is defined. This leads to consistent estimates of the parameters of the  $L$  time slice version of the simple branching diffusion process. Under some conditions  $A \uparrow \mathbb{R}^d$ , limiting normal distributions are obtained for these estimates.

The parameters of an  $L$  time slice version of the simple branching diffusion process are time scale dependent. If  $L = 2$ , with times  $\tau$  and  $\tau + t$ , where  $t > 0$  is an experimental parameter, then even the age,  $\tau$ , of the process can be estimated. Since the process is observed on a large set  $A \subseteq \mathbb{R}^d$  at the fixed, unknown times, this means that with no "tagging" or following of the individuals, such parameters as the rate of movement (diffusion rate) and the age,  $\tau$ , can still be estimated.

## Themes:

- Brillinger-Type Mixing/Time Series
- CLT
- Fourier Transforms
- Asymptotic Likelihood
- Parameter Identifiability



## Early Career

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**1978: NSERC Postdoctoral fellowship in Berkeley with David Brillinger.**

**Early 1980s: McMaster University**

**1982: UWO**

**1988: First PhD Student - supposed to work on statistical inference for point processes. RJK: “Read these papers by Brillinger and we’ll work on this CLT for cluster point processes. It should be a pretty straightforward application of an  $m$ -dependent CLT.”**

**1988-89: Study Leave in Heidelberg, interacting with P. Tautu and W. Rittgen - interacting particle systems as models for tumour growth**

## Tips for PhD Supervision

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**Fall, 1989: First PhD Student - still supposed to work on statistical inference for point processes - little progress**

**RJK: “OK, but let’s not be at this same point next year.”**

## **Tips for PhD Supervision - Part II**

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**Fall, 1990: First PhD Student - still supposed to work on statistical inference for point processes - still little progress**

**... but some progress on statistical identifiability of parameters of an interacting particle system as a model for tumour growth**

**RJK: “OK, but let’s not be at this same point next year.”**

## **Tips for PhD Supervision - Part III**

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**Fall, 1991: First PhD Student - still supposed to work on statistical inference for point processes - still little progress**

**RJK: “OK, I guess the thesis is on interacting particle systems. ”**

## Consequences

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**From Reg, I have learned that all areas of mathematics and statistics have points of interest.**

**With Reg, I have worked on many problems that he inspired: interacting particle systems, first for tumour growth, but more successfully, for fire growth; time series bootstrapping, point process bootstrapping\*; parameter identifiability of interacting particle system models; coupling proofs; moment-based differential equations; ...**

\*I finally got this figured out, but see final slides

## **My Own Problems with Point Processes**

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**The papers by Brillinger on point processes are sometimes a bit terse, and I had a lot of trouble putting them into a context that I could understand. I struggled unsuccessfully to decipher inference for point process intensity functions.**

**Shortly after arriving at my first job at the University of Winnipeg, a psychology professor called me up and asked me if I knew anything about point processes.**



## **My Own Problems with Point Processes**

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**I could not tell the truth.**

**WJB: “Sure, what do you need to know?”**

## **My Own Problems with Point Processes**

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**UWpg Psychologist: “There are these papers by Brillinger ... ”**

## Epilogue

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**With Jennifer Asimit, my own first PhD student, I received the CJS Best Paper Award in 2006 for**

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**Asimit, J. and Braun, W.J. Third order point process intensity estimation for reaction time experiment data. *Can. J. Statist.* 33 243–257, 2005.**

**Thanks, Reg!**